Power analysis

Mikhail Dozmorov

Contents

Example .......................... 1
  Components of power analysis ................................ 1
  Hypotheses ......................................................... 1
  At what point on $H_0$ distribution does rejection region start (corresponds to $\alpha/2$)? .... 1
  $H_0$ distribution ...................................................... 2
  $H_A$ distribution ...................................................... 2
  How to find $\beta$? ..................................................... 2
  References .......................................................... 3

Example

You are working on designing a new cancer drug and want to increase its half life in the body from current standard of 20 hours (with a std. dev. of 4 hours) for cancer drugs.

You believe your new design can increase the half-life by 2 hours to 22 hours. You have 44 random samples of your new drug for half life.

Determine the power of your study (for $\alpha = 0.05$). If it is lacking power, what can you do to increase its power?

Components of power analysis

- Model (test)
- Test Effect (effect size and variability)
- Sample size ($n$)
- Test size (significance level, $\alpha$)
- Power of test ($1 - \beta$)

Hypotheses

- $H_0 : \mu = 20$
- $H_A : \mu = 22$

Effect size $22-20=2$

Two-tailed t-test

At what point on $H_0$ distribution does rejection region start (corresponds to $\alpha/2$)?

T-table for df=43. $t_{critical\ value} = 2.0166922$

$$t_{stat} = \frac{\hat{x} - \mu}{s / \sqrt{n}}$$
\[ 2.0166922 = \frac{x_{\text{critical}} - 20}{4/\sqrt{44}} \]

\[ x_{\text{critical}} = 21.2161 \]

**H₀ distribution**

Area to the right corresponds to \( \alpha/2 \)

\[ H_A \text{ distribution} \]

Area to the right corresponds to \( AUC = \beta \) - probability of a Type II error.

To the right - \( 1 - \beta \) - power

**How to find \( \beta \)?**

What is the probability of having a value less than \( t_{\text{critical}} \) (21.2161) in \( H_A \) distribution?

\[ t_\beta = (21.2161 - 22)/(4/\sqrt{44}) = -1.299 \]
Plot this value on a t-distribution

\[
\beta = P(t < -1.299) = P(t > 1.299) = 0.096972 \ (0.1003 \text{ in the video})
\]

\[
Power = 1 - \beta = 1 - 0.1003 = 0.8997
\]

Power > 0.8, therefore experimental design is powerful. Should be able to detect effect size.

References

See power analysis in action: “Power analysis example” video https://www.youtube.com/watch?v=fRm2dEWSJrk by Matthew Novak.

- http://www.3rs-reduction.co.uk/html/6__power_and_sample_size.html
- http://www.ats.ucla.edu/stat/seminars/Intro_power/
- http://biostat.mc.vanderbilt.edu/wiki/Main/PowerSampleSize

Online tools

- http://powerandsamplesize.com/
- http://www.sample-size.net/sample-size-survival-analysis/
- https://www.stat.ubc.ca/~rollin/stats/ssize/n2.html
- http://scotty.genetics.utah.edu/

R packages


- **FDRsampsize** - Compute Sample Size that Meets Requirements for Average Power and FDR, https://cran.r-project.org/web/packages/FDRsampsize/index.html
