

Example 2 Suppose you are given the data below in a two by two table.

```
> pm.ex<-pm(replicates,featureNames(replicates)[grep("205586_x_at",featureNames(replicates))])
> pm.ex
```

	REP1.CEL	REP2.CEL	REP3.CEL
205586_x_at1	134.3	110.3	138.0
205586_x_at2	265.0	304.5	290.3
205586_x_at3	254.3	277.8	285.3
205586_x_at4	73.5	86.3	73.8
205586_x_at5	145.0	157.5	179.3
205586_x_at6	202.0	209.0	207.5
205586_x_at7	228.3	237.3	224.5
205586_x_at8	319.3	343.3	405.8
205586_x_at9	829.5	736.5	820.0
205586_x_at10	102.5	118.3	122.8
205586_x_at11	138.3	96.3	103.0

(0) iteration : Start with the raw data in a two-way table, I represents the row effect, J represent the column effect.

I	J		
	1	2	3
1	134.3	110.3	138.0
2	265.0	304.5	290.3
3	254.3	277.8	285.3
4	73.5	86.3	73.8
5	145.0	157.5	179.3
6	202.0	209.0	207.5
7	228.3	237.3	224.5
8	319.3	343.3	405.8
9	829.5	736.5	820.0
10	102.5	118.3	122.8
11	138.3	96.3	102.0

(1) iteration, step a: The previous row $a_i^{(0)}$, column $b_j^{(0)}$, and main effect $m^{(0)}$ are initialized to 0. Then, find the median of each row, $\Delta a_i^{(1)}$.

I	J			Row median $\Delta a_i^{(1)}$	Previous row effect $a_i^{(0)}$
	1	2	3		
1	134.3	110.3	138.0	134.3	0
2	265.0	304.5	290.3	290.3	0
3	254.3	277.8	285.3	277.8	0
4	73.5	86.3	73.8	73.8	0
5	145.0	157.5	179.3	157.5	0
6	202.0	209.0	207.5	207.5	0
7	228.3	237.3	224.5	228.3	0
8	319.3	343.3	405.8	343.3	0
9	829.5	736.5	820.0	820.0	0
10	102.5	118.3	122.8	118.3	0
11	138.3	96.3	103.0	103.0	0
Prev Column Effect $b_j^{(0)}$	0	0	0		$m^{(0)}=0$

(1) iteration, step b: Row polish by subtracting the row median values from the corresponding row observations. Find the column medians after the row polish, $\Delta b_j^{(1)}$.

I	J			Row median $\Delta a_i^{(1)}$	Previous row effect $a_i^{(0)}$
	1	2	3		
1	0	-24.0	3.7	134.3	0
2	-25.3	14.2	0	290.3	0
3	-23.5	0	7.5	277.8	0
4	-0.3	12.5	0	73.8	0
5	-12.5	0	21.8	157.5	0
6	-5.5	1.5	0	207.5	0
7	0	9.0	-3.8	228.3	0
8	-24.0	0	62.5	343.3	0
9	9.5	-83.5	0	820.0	0
10	-15.8	0	4.5	118.3	0
11	35.3	-6.7	0	103.0	0
Column median $\Delta b_j^{(1)}$	-5.5	0	0		
Prev Column Effect $b_j^{(0)}$	0	0	0	0	$m^{(0)}=0$

(1) iteration, step c: Column polish by subtracting the column median values from the corresponding column observations.

I	J			Row median $\Delta a_i^{(1)}$	Previous row effect $a_i^{(0)}$
	1	2	3		
1	5.5	-24.0	3.7	134.3	0
2	-19.8	14.2	0.0	290.3	0
3	-18.0	0.0	7.5	277.8	0
4	5.2	12.5	0.0	73.8	0
5	-7.0	0.0	21.8	157.5	0
6	0.0	1.5	0.0	207.5	0
7	5.5	9.0	-3.8	228.3	0
8	-18.5	0.0	62.5	343.3	0
9	15.0	-83.5	0.0	820.0	0
10	-10.3	0.0	4.5	118.3	0
11	40.8	-6.7	0.0	102.0	0
Column median $\Delta b_j^{(1)}$	-5.5	0	0		
Prev Column Effect $b_j^{(0)}$	0	0	0		$m^{(0)}=0$

(1) iteration, step d: Estimate the effects by

$$\Delta m_a^{(1)} = \text{median}(a_i^{(0)} + \Delta a_i^{(1)}) = 207.5$$

$$\Delta m_b^{(1)} = \text{median}(b_j^{(0)}) = 0$$

$$m^{(1)} = m^{(0)} + \Delta m_a^{(1)} + \Delta m_b^{(1)} = 0 + 207.5 + 0 = 207.5 \quad (\text{overall effect estimate after 1st iteration})$$

$$a_i^{(1)} = a_i^{(0)} + \Delta a_i^{(1)} - \Delta m_a^{(1)} \quad (\text{row effect estimates after 1st iteration})$$

$$b_j^{(1)} = b_j^{(0)} + \Delta b_j^{(1)} - \Delta m_b^{(1)} \quad (\text{column effect estimates after 1st iteration})$$

I	J			Row median $\Delta a_i^{(1)}$	Previous row effect $a_i^{(0)}$	$a_i^{(0)} + \Delta a_i^{(1)}$	$a_i^{(1)} = a_i^{(0)} + \Delta a_i^{(1)} - \Delta m_a^{(1)}$
	1	2	3				
1	5.5	-24.0	3.7	134.3	0	134.3	-73.2
2	-19.8	14.2	0.0	290.3	0	290.3	82.8
3	-18.0	0.0	7.5	277.8	0	277.8	70.3
4	5.2	12.5	0.0	73.8	0	73.8	-133.7
5	-7.0	0.0	21.8	157.5	0	157.5	-50.0
6	0.0	1.5	0.0	207.5	0	207.5	0.0
7	5.5	9.0	-3.8	228.3	0	228.3	20.8
8	-18.5	0.0	62.5	343.3	0	343.3	135.8
9	15.0	-83.5	0.0	820.0	0	820.0	612.5
10	-10.3	0.0	4.5	118.3	0	118.3	-89.2
11	40.8	-6.7	0.0	102.0	0	102.0	-104.5
Column median $\Delta b_j^{(1)}$	-5.5	0	0				
Prev Column Effect $b_j^{(0)}$	0	0	0		$m^{(0)} = 0$		
$b_j^{(1)} = b_j^{(0)} + \Delta b_j^{(1)} - \Delta m_b^{(1)}$	-5.5	0	0				

(2) **iteration, step a:** For the second iteration, retain the previous row $a_i^{(1)}$, column $b_j^{(1)}$, and main effect $m^{(1)}$ as estimates of row and column and main effects. Then, find the median of each row.

I	J			Row median $\Delta a_i^{(2)}$	Previous row effect $a_i^{(1)}$
	1	2	3		
1	5.5	-24.0	3.7	3.7	-73.2
2	-19.8	14.2	0.0	0.0	82.8
3	-18.0	0.0	7.5	0.0	70.3
4	5.2	12.5	0.0	5.2	-133.7
5	-7.0	0.0	21.8	0.0	-50.0
6	0.0	1.5	0.0	0.0	0.0
7	5.5	9.0	-3.8	5.5	20.8
8	-18.5	0.0	62.5	0.0	135.8
9	15.0	-83.5	0.0	0.0	612.5
10	-10.3	0.0	4.5	0.0	-89.2
11	40.8	-6.7	0.0	0.0	-104.5
Prev Column Effect $b_j^{(1)}$	-5.5	0	0		$m^{(1)}=207.5$

(2) iteration, step b: Row polish by subtracting the row median values from the corresponding row observations. Find the column medians after the row polish.

I	J			Row median $\Delta a_i^{(2)}$	Previous row effect $a_i^{(1)}$
	1	2	3		
1	1.8	-27.7	0.0	3.7	-73.2
2	-19.8	14.2	0.0	0.0	82.8
3	-18.0	0.0	7.5	0.0	70.3
4	0.0	7.3	-5.2	5.2	-133.7
5	-7.0	0.0	21.8	0.0	-50.0
6	0.0	1.5	0.0	0.0	0.0
7	0.0	3.5	-9.3	5.5	20.8
8	-18.5	0.0	62.5	0.0	135.8
9	15.0	-83.5	0.0	0.0	612.5
10	-10.3	0.0	4.5	0.0	-89.2
11	40.8	-6.7	0.0	0.0	-104.5
Column median $\Delta b_j^{(2)}$	0	0	0		
Prev Column Effect $b_j^{(1)}$	-5.5	0	0	0	$m^{(1)}=207.5$

(2) iteration, step c: Column polish by subtracting the column median values from the corresponding column observations.

(2) iteration, step d: Estimate the effects by

$$\Delta m_a^{(2)} = \text{median}(a_i^{(1)} + \Delta a_i^{(2)}) = 0$$

$$\Delta m_b^{(2)} = \text{median}(b_j^{(1)}) = 0$$

$$m^{(2)} = m^{(1)} + \Delta m_a^{(2)} + \Delta m_b^{(2)} = 207.5 + 0 + 0 = 207.5 \quad (\text{overall effect estimate after 2nd iteration})$$

$$a_i^{(2)} = a_i^{(1)} + \Delta a_i^{(2)} - \Delta m_a^{(2)} \quad (\text{row effect estimates after 1st iteration})$$

$$b_j^{(2)} = b_j^{(1)} + \Delta b_j^{(2)} - \Delta m_b^{(2)} \quad (\text{column effect estimates after 1st iteration})$$

I	J			Row median $\Delta a_i^{(2)}$	Previous row effect $a_i^{(1)}$	$a_i^{(1)} + \Delta a_i^{(2)}$	$a_i^{(2)} = a_i^{(1)} + \Delta a_i^{(2)} - \Delta m_a^{(2)}$
	1	2	3				
1	1.8	-27.7	0.0	3.7	-73.2	-69.5	-69.5
2	-19.8	14.2	0.0	0.0	82.8	82.8	82.8
3	-18.0	0.0	7.5	0.0	70.3	70.3	70.3
4	0.0	7.3	-5.2	5.2	-133.7	-128.5	-128.5
5	-7.0	0.0	21.8	0.0	-50.0	-50.0	-50.0
6	0.0	1.5	0.0	0.0	0.0	0.0	0.0
7	0.0	3.5	-9.3	5.5	20.8	26.3	26.3
8	-18.5	0.0	62.5	0.0	135.8	135.8	135.8
9	15.0	-83.5	0.0	0.0	612.5	612.5	612.5
10	-10.3	0.0	4.5	0.0	-89.2	-89.2	-89.2
11	40.8	-6.7	0.0	0.0	-104.5	-104.5	-104.5
Column median $\Delta b_j^{(2)}$	0	0	0				
Prev Column Effect $b_j^{(1)}$	-5.5	0	0		$m^{(1)} = 207.5$		
$b_j^{(2)} = b_j^{(1)} + \Delta b_j^{(2)} - \Delta m_b^{(2)}$	-5.5	0	0				

Note that summing the main effect, row effect, column effect, and residual yields the original observation $y_{11,2} = 207.5 - 104.5 + 0 - 6.7 = 96.3$. The intensity for chips 1, 2, and 3 would be $207.5 - 5.5 = 202$; $207.5 - 0 = 207.5$; and $207.5 - 0 = 207.5$ respectively. The log2 intensities would be $\log_2(202) = 7.658212$ for chip 1 and $\log_2(207.5) = 7.696968$ for chips 2 and 3.