

(0) iteration : Start with the raw data in a two-way table

| I | J | | | |
|---|-----------------------|-----------------------|-----------------------|-----------------------|
| | 1 | 2 | 3 | 4 |
| 1 | $e_{11}^{(0)}=y_{11}$ | $e_{12}^{(0)}=y_{12}$ | $e_{13}^{(0)}=y_{13}$ | $e_{14}^{(0)}=y_{14}$ |
| 2 | $e_{21}^{(0)}=y_{21}$ | $e_{22}^{(0)}=y_{22}$ | $e_{23}^{(0)}=y_{23}$ | $e_{24}^{(0)}=y_{24}$ |
| 3 | $e_{31}^{(0)}=y_{31}$ | $e_{32}^{(0)}=y_{32}$ | $e_{33}^{(0)}=y_{33}$ | $e_{34}^{(0)}=y_{34}$ |

(1) iteration, step a: Find the median of each row $\Delta a_i^{(1)} = \text{median}(e_{i1}^{(0)}, \dots, e_{iJ}^{(0)})$ and $\Delta m_b^{(1)} = \text{median}(b_j^{(0)}\text{'s})$

| I | J | | | | New med | Prev Effect |
|-------------|-----------------------|-----------------------|-----------------------|-----------------------|---|-----------------|
| | 1 | 2 | 3 | 4 | | |
| 1 | $e_{11}^{(0)}=y_{11}$ | $e_{12}^{(0)}=y_{12}$ | $e_{13}^{(0)}=y_{13}$ | $e_{14}^{(0)}=y_{14}$ | $\Delta a_1^{(1)} = \text{median}(e_{11}^{(0)}, \dots, e_{1J}^{(0)})$ | $a_1^{(0)} = 0$ |
| 2 | $e_{21}^{(0)}=y_{21}$ | $e_{22}^{(0)}=y_{22}$ | $e_{23}^{(0)}=y_{23}$ | $e_{24}^{(0)}=y_{24}$ | $\Delta a_2^{(1)} = \text{median}(e_{21}^{(0)}, \dots, e_{2J}^{(0)})$ | $a_2^{(0)} = 0$ |
| 3 | $e_{31}^{(0)}=y_{31}$ | $e_{32}^{(0)}=y_{32}$ | $e_{33}^{(0)}=y_{33}$ | $e_{34}^{(0)}=y_{34}$ | $\Delta a_3^{(1)} = \text{median}(e_{31}^{(0)}, \dots, e_{3J}^{(0)})$ | $a_3^{(0)} = 0$ |
| Prev Effect | $b_1^{(0)} = 0$ | $b_2^{(0)} = 0$ | $b_3^{(0)} = 0$ | $b_4^{(0)} = 0$ | $\Delta m_b^{(1)} = \text{median}(b_j^{(0)}\text{'s}) = 0$ | $m^{(0)} = 0$ |

(1) iteration, step b: Row polish by subtracting the $\Delta a_i^{(1)} = \text{median}(e_{i1}^{(0)}, \dots, e_{iJ}^{(0)})$ values.

| I | J | | | | New med | Prev Effect |
|-------------|--|--|--|--|---|-----------------|
| | 1 | 2 | 3 | 4 | | |
| 1 | $d_{11}^{(1)} = e_{11}^{(0)} - \Delta a_1^{(1)}$ | $d_{12}^{(1)} = e_{12}^{(0)} - \Delta a_1^{(1)}$ | $d_{13}^{(1)} = e_{13}^{(0)} - \Delta a_1^{(1)}$ | $d_{14}^{(1)} = e_{14}^{(0)} - \Delta a_1^{(1)}$ | $\Delta a_1^{(1)} = \text{median}(e_{11}^{(0)}, \dots, e_{1J}^{(0)})$ | $a_1^{(0)} = 0$ |
| 2 | $d_{21}^{(1)} = e_{21}^{(0)} - \Delta a_2^{(1)}$ | $d_{22}^{(1)} = e_{22}^{(0)} - \Delta a_2^{(1)}$ | $d_{23}^{(1)} = e_{23}^{(0)} - \Delta a_2^{(1)}$ | $d_{24}^{(1)} = e_{24}^{(0)} - \Delta a_2^{(1)}$ | $\Delta a_2^{(1)} = \text{median}(e_{21}^{(0)}, \dots, e_{2J}^{(0)})$ | $a_2^{(0)} = 0$ |
| 3 | $d_{31}^{(1)} = e_{31}^{(0)} - \Delta a_3^{(1)}$ | $d_{32}^{(1)} = e_{32}^{(0)} - \Delta a_3^{(1)}$ | $d_{33}^{(1)} = e_{33}^{(0)} - \Delta a_3^{(1)}$ | $d_{34}^{(1)} = e_{34}^{(0)} - \Delta a_3^{(1)}$ | $\Delta a_3^{(1)} = \text{median}(e_{31}^{(0)}, \dots, e_{3J}^{(0)})$ | $a_3^{(0)} = 0$ |
| Prev Effect | $b_1^{(0)} = 0$ | $b_2^{(0)} = 0$ | $b_3^{(0)} = 0$ | $b_4^{(0)} = 0$ | $\Delta m_b^{(1)} = \text{median}(b_j^{(0)}\text{'s}) = 0$ | $m^{(0)} = 0$ |

(1) iteration, step c: Find the median of each column $\Delta b_j^{(1)} = \text{med}(d_{1j}^{(1)}, \dots, d_{lj}^{(1)})$ and calculate $\Delta m_a^{(1)} = \text{med}(a_i^{(0)} + \Delta a_i^{(1)})$

| I | J | | | | New med | Prev |
|--------------|--|--|--|--|---|-----------------|
| | 1 | 2 | 3 | 4 | | |
| 1 | $d_{11}^{(1)} = e_{11}^{(0)} - \Delta a_1^{(1)}$ | $d_{12}^{(1)} = e_{12}^{(0)} - \Delta a_1^{(1)}$ | $d_{13}^{(1)} = e_{13}^{(0)} - \Delta a_1^{(1)}$ | $d_{14}^{(1)} = e_{14}^{(0)} - \Delta a_1^{(1)}$ | $\Delta a_1^{(1)} = \text{median}(e_{11}^{(0)}, \dots, e_{1J}^{(0)})$ | $a_1^{(0)} = 0$ |
| 2 | $d_{21}^{(1)} = e_{21}^{(0)} - \Delta a_2^{(1)}$ | $d_{22}^{(1)} = e_{22}^{(0)} - \Delta a_2^{(1)}$ | $d_{23}^{(1)} = e_{23}^{(0)} - \Delta a_2^{(1)}$ | $d_{24}^{(1)} = e_{24}^{(0)} - \Delta a_2^{(1)}$ | $\Delta a_2^{(1)} = \text{median}(e_{21}^{(0)}, \dots, e_{2J}^{(0)})$ | $a_2^{(0)} = 0$ |
| 3 | $d_{31}^{(1)} = e_{31}^{(0)} - \Delta a_3^{(1)}$ | $d_{32}^{(1)} = e_{32}^{(0)} - \Delta a_3^{(1)}$ | $d_{33}^{(1)} = e_{33}^{(0)} - \Delta a_3^{(1)}$ | $d_{34}^{(1)} = e_{34}^{(0)} - \Delta a_3^{(1)}$ | $\Delta a_3^{(1)} = \text{median}(e_{31}^{(0)}, \dots, e_{3J}^{(0)})$ | $a_3^{(0)} = 0$ |
| New med | $\Delta b_1^{(1)} = \text{med}(d_{11}^{(1)}, \dots, d_{l1}^{(1)})$ | $\Delta b_2^{(1)} = \text{med}(d_{12}^{(1)}, \dots, d_{l2}^{(1)})$ | $\Delta b_3^{(1)} = \text{med}(d_{13}^{(1)}, \dots, d_{l3}^{(1)})$ | $\Delta b_4^{(1)} = \text{med}(d_{14}^{(1)}, \dots, d_{l4}^{(1)})$ | $\Delta m_a^{(1)} = \text{med}(a_i^{(0)} + \Delta a_i^{(1)})$ | |
| Prev Effects | $b_1^{(0)} = 0$ | $b_2^{(0)} = 0$ | $b_3^{(0)} = 0$ | $b_4^{(0)} = 0$ | $\Delta m_b^{(1)} = \text{median}(b_j^{(0)}\text{'s}) = 0$ | |

Estimate the effects by

$$m^{(1)} = m^{(0)} + \Delta m_a^{(1)} + \Delta m_b^{(1)}$$

$$a_i^{(1)} = a_i^{(0)} + \Delta a_i^{(1)} - \Delta m_a^{(1)}$$

$$b_j^{(1)} = b_j^{(0)} + \Delta b_j^{(1)} - \Delta m_b^{(1)}$$

(1) iteration, step d: The cell values are updated by subtracting $\Delta b_j^{(1)} = \text{med}(d_{1j}^{(1)}, \dots, d_{lj}^{(1)})$

| I | J | | | | Prev Effects |
|--------------|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | |
| 1 | $e_{11}^{(1)} = d_{11}^{(1)} - \Delta b_1^{(1)}$ | $e_{12}^{(1)} = d_{12}^{(1)} - \Delta b_2^{(1)}$ | $e_{13}^{(1)} = d_{13}^{(1)} - \Delta b_3^{(1)}$ | $e_{14}^{(1)} = d_{14}^{(1)} - \Delta b_4^{(1)}$ | $a_1^{(1)} = a_1^{(0)} + \Delta a_1^{(1)} - \Delta m_a^{(1)}$ |
| 2 | $e_{21}^{(1)} = d_{21}^{(1)} - \Delta b_1^{(1)}$ | $e_{22}^{(1)} = d_{22}^{(1)} - \Delta b_2^{(1)}$ | $e_{23}^{(1)} = d_{23}^{(1)} - \Delta b_3^{(1)}$ | $e_{24}^{(1)} = d_{24}^{(1)} - \Delta b_4^{(1)}$ | $a_2^{(1)} = a_2^{(0)} + \Delta a_2^{(1)} - \Delta m_a^{(1)}$ |
| 3 | $e_{31}^{(1)} = d_{31}^{(1)} - \Delta b_1^{(1)}$ | $e_{32}^{(1)} = d_{32}^{(1)} - \Delta b_2^{(1)}$ | $e_{33}^{(1)} = d_{33}^{(1)} - \Delta b_3^{(1)}$ | $e_{34}^{(1)} = d_{34}^{(1)} - \Delta b_4^{(1)}$ | $a_3^{(1)} = a_3^{(0)} + \Delta a_3^{(1)} - \Delta m_a^{(1)}$ |
| Prev Effects | $b_1^{(1)} = b_1^{(0)} + \Delta b_1^{(1)} - \Delta m_b^{(1)}$ | $b_2^{(1)} = b_2^{(0)} + \Delta b_2^{(1)} - \Delta m_b^{(1)}$ | $b_3^{(1)} = b_3^{(0)} + \Delta b_3^{(1)} - \Delta m_b^{(1)}$ | $b_4^{(1)} = b_4^{(0)} + \Delta b_4^{(1)} - \Delta m_b^{(1)}$ | $m^{(1)} = m^{(0)} + \Delta m_a^{(1)} + \Delta m_b^{(1)}$ |

(2) iteration, step a: row polish by finding the median of each row

| I | J | | | | New med | Prev Effects |
|--------------|---|---|---|---|--|---|
| | 1 | 2 | 3 | 4 | | |
| 1 | $e_{11}^{(1)} = d_{11}^{(1)} - \Delta b_1^{(1)}$ | $e_{12}^{(1)} = d_{12}^{(1)} - \Delta b_2^{(1)}$ | $e_{13}^{(1)} = d_{13}^{(1)} - \Delta b_3^{(1)}$ | $e_{14}^{(1)} = d_{14}^{(1)} - \Delta b_4^{(1)}$ | $\Delta a_1^{(2)} = \text{med}(e_{11}^{(1)}, \dots, e_{1J}^{(1)})$ | $a_1^{(1)} = a_1^{(0)} + \Delta a_1^{(1)} - \Delta m_a^{(1)}$ |
| 2 | $e_{21}^{(1)} = d_{21}^{(1)} - \Delta b_1^{(1)}$ | $e_{22}^{(1)} = d_{22}^{(1)} - \Delta b_2^{(1)}$ | $e_{23}^{(1)} = d_{23}^{(1)} - \Delta b_3^{(1)}$ | $e_{24}^{(1)} = d_{24}^{(1)} - \Delta b_4^{(1)}$ | $\Delta a_2^{(2)} = \text{med}(e_{21}^{(1)}, \dots, e_{2J}^{(1)})$ | $a_2^{(1)} = a_2^{(0)} + \Delta a_2^{(1)} - \Delta m_a^{(1)}$ |
| 3 | $e_{31}^{(1)} = d_{31}^{(1)} - \Delta b_1^{(1)}$ | $e_{32}^{(1)} = d_{32}^{(1)} - \Delta b_2^{(1)}$ | $e_{33}^{(1)} = d_{33}^{(1)} - \Delta b_3^{(1)}$ | $e_{34}^{(1)} = d_{34}^{(1)} - \Delta b_4^{(1)}$ | $\Delta a_3^{(2)} = \text{med}(e_{31}^{(1)}, \dots, e_{3J}^{(1)})$ | $a_3^{(1)} = a_3^{(0)} + \Delta a_3^{(1)} - \Delta m_a^{(1)}$ |
| Prev Effects | $b_1^{(1)} = b_1^{(0)} + \Delta b_1^{(1)} - \Delta m_b^{(1)}$ | $b_2^{(1)} = b_2^{(0)} + \Delta b_2^{(1)} - \Delta m_b^{(1)}$ | $b_3^{(1)} = b_3^{(0)} + \Delta b_3^{(1)} - \Delta m_b^{(1)}$ | $b_4^{(1)} = b_4^{(0)} + \Delta b_4^{(1)} - \Delta m_b^{(1)}$ | $\Delta m_b^{(2)} = \text{med}(b_j^{(1)}\text{'s})$ | $m^{(1)} = m^{(0)} + \Delta m_a^{(1)} + \Delta m_b^{(1)}$ |

(2) iteration, step b: Next, the cell values are updated by subtracting the $\Delta a_1^{(2)} = \text{median}(e_{11}^{(1)}, \dots, e_{1J}^{(1)})$ values.

| I | J | | | | New med | Prev Effects |
|--------------|---|---|---|---|--|---|
| | 1 | 2 | 3 | 4 | | |
| 1 | $d_{11}^{(2)} = e_{11}^{(1)} - \Delta a_1^{(2)}$ | $d_{12}^{(2)} = e_{12}^{(1)} - \Delta a_1^{(2)}$ | $d_{13}^{(2)} = e_{13}^{(1)} - \Delta a_1^{(2)}$ | $d_{14}^{(2)} = e_{14}^{(1)} - \Delta a_1^{(2)}$ | $\Delta a_1^{(2)} = \text{med}(e_{11}^{(1)}, \dots, e_{1J}^{(1)})$ | $a_1^{(1)} = a_1^{(0)} + \Delta a_1^{(1)} - \Delta m_a^{(1)}$ |
| 2 | $d_{21}^{(2)} = e_{21}^{(1)} - \Delta a_2^{(2)}$ | $d_{22}^{(2)} = e_{22}^{(1)} - \Delta a_2^{(2)}$ | $d_{23}^{(2)} = e_{23}^{(1)} - \Delta a_2^{(2)}$ | $d_{24}^{(2)} = e_{24}^{(1)} - \Delta a_2^{(2)}$ | $\Delta a_2^{(2)} = \text{med}(e_{21}^{(1)}, \dots, e_{2J}^{(1)})$ | $a_2^{(1)} = a_2^{(0)} + \Delta a_2^{(1)} - \Delta m_a^{(1)}$ |
| 3 | $d_{31}^{(2)} = e_{31}^{(1)} - \Delta a_3^{(2)}$ | $d_{32}^{(2)} = e_{32}^{(1)} - \Delta a_3^{(2)}$ | $d_{33}^{(2)} = e_{33}^{(1)} - \Delta a_3^{(2)}$ | $d_{34}^{(2)} = e_{34}^{(1)} - \Delta a_3^{(2)}$ | $\Delta a_3^{(2)} = \text{med}(e_{31}^{(1)}, \dots, e_{3J}^{(1)})$ | $a_3^{(1)} = a_3^{(0)} + \Delta a_3^{(1)} - \Delta m_a^{(1)}$ |
| Prev Effects | $b_1^{(1)} = b_1^{(0)} + \Delta b_1^{(1)} - \Delta m_b^{(1)}$ | $b_2^{(1)} = b_2^{(0)} + \Delta b_2^{(1)} - \Delta m_b^{(1)}$ | $b_3^{(1)} = b_3^{(0)} + \Delta b_3^{(1)} - \Delta m_b^{(1)}$ | $b_4^{(1)} = b_4^{(0)} + \Delta b_4^{(1)} - \Delta m_b^{(1)}$ | $\Delta m_b^{(2)} = \text{med}(b_j^{(1)}\text{'s})$ | $m^{(1)} = m^{(0)} + \Delta m_a^{(1)} + \Delta m_b^{(1)}$ |

(2) iteration, step c: column polish by finding the median of each column

| I | J | | | | New med | Prev Effects |
|--------------|--|--|--|--|---|--------------|
| | 1 | 2 | 3 | 4 | | |
| 1 | $d_{11}^{(2)} = e_{11}^{(1)} - \Delta a_1^{(2)}$ | $d_{12}^{(2)} = e_{12}^{(1)} - \Delta a_1^{(2)}$ | $d_{13}^{(2)} = e_{13}^{(1)} - \Delta a_1^{(2)}$ | $d_{14}^{(2)} = e_{14}^{(1)} - \Delta a_1^{(2)}$ | $\Delta a_1^{(2)} = \text{median}(e_{11}^{(1)}, \dots, e_{1J}^{(1)})$ | $a_1^{(1)}$ |
| 2 | $d_{21}^{(2)} = e_{21}^{(1)} - \Delta a_2^{(2)}$ | $d_{22}^{(2)} = e_{22}^{(1)} - \Delta a_2^{(2)}$ | $d_{23}^{(2)} = e_{23}^{(1)} - \Delta a_2^{(2)}$ | $d_{24}^{(2)} = e_{24}^{(1)} - \Delta a_2^{(2)}$ | $\Delta a_2^{(2)} = \text{median}(e_{21}^{(1)}, \dots, e_{2J}^{(1)})$ | $a_2^{(1)}$ |
| 3 | $d_{31}^{(2)} = e_{31}^{(1)} - \Delta a_3^{(2)}$ | $d_{32}^{(2)} = e_{32}^{(1)} - \Delta a_3^{(2)}$ | $d_{33}^{(2)} = e_{33}^{(1)} - \Delta a_3^{(2)}$ | $d_{34}^{(2)} = e_{34}^{(1)} - \Delta a_3^{(2)}$ | $\Delta a_3^{(2)} = \text{median}(e_{31}^{(1)}, \dots, e_{3J}^{(1)})$ | $a_3^{(1)}$ |
| New med | $\Delta b_1^{(2)} = \text{med}(d_{11}^{(2)}, \dots, d_{I1}^{(2)})$ | $\Delta b_2^{(2)} = \text{med}(d_{12}^{(2)}, \dots, d_{I2}^{(2)})$ | $\Delta b_3^{(2)} = \text{med}(d_{13}^{(2)}, \dots, d_{I3}^{(2)})$ | $\Delta b_4^{(2)} = \text{med}(d_{14}^{(2)}, \dots, d_{I4}^{(2)})$ | $\Delta m_a^{(2)} = \text{med}(a_i^{(1)} + \Delta a_i^{(2)})$ | |
| Prev Effects | $b_1^{(1)}$ | $b_2^{(1)}$ | $b_3^{(1)}$ | $b_4^{(1)}$ | $\Delta m_b^{(2)} = \text{median}(b_j^{(1)}\text{'s})$ | $m^{(1)}$ |

Estimate the effects by

$$m^{(2)} = m^{(1)} + \Delta m_a^{(2)} + \Delta m_b^{(2)}$$

$$a_i^{(2)} = a_i^{(1)} + \Delta a_i^{(2)} - \Delta m_a^{(2)}$$

$$b_j^{(2)} = b_j^{(1)} + \Delta b_j^{(2)} - \Delta m_b^{(2)}$$

(2) iteration, step d: The cell values are updated by subtracting $\Delta b_j^{(2)} = \text{med}(d_{1j}^{(2)}, \dots, d_{Ij}^{(2)})$. $m^{(2)}$ is the main effect, $a_i^{(2)}$ are the row effects, $b_j^{(2)}$ are the column effects, and $e_{ij}^{(2)}$ are the residuals.

| I | J | | | | Prev Effects |
|--------------|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | |
| 1 | $e_{11}^{(2)} = d_{11}^{(2)} - \Delta b_1^{(2)}$ | $e_{12}^{(2)} = d_{12}^{(2)} - \Delta b_2^{(2)}$ | $e_{13}^{(2)} = d_{13}^{(2)} - \Delta b_3^{(2)}$ | $e_{14}^{(2)} = d_{14}^{(2)} - \Delta b_4^{(2)}$ | $a_1^{(2)} = a_1^{(1)} + \Delta a_1^{(2)} - \Delta m_a^{(2)}$ |
| 2 | $e_{21}^{(2)} = d_{21}^{(2)} - \Delta b_1^{(2)}$ | $e_{22}^{(2)} = d_{22}^{(2)} - \Delta b_2^{(2)}$ | $e_{23}^{(2)} = d_{23}^{(2)} - \Delta b_3^{(2)}$ | $e_{24}^{(2)} = d_{24}^{(2)} - \Delta b_4^{(2)}$ | $a_2^{(2)} = a_2^{(1)} + \Delta a_2^{(2)} - \Delta m_a^{(2)}$ |
| 3 | $e_{31}^{(2)} = d_{31}^{(2)} - \Delta b_1^{(2)}$ | $e_{32}^{(2)} = d_{32}^{(2)} - \Delta b_2^{(2)}$ | $e_{33}^{(2)} = d_{33}^{(2)} - \Delta b_3^{(2)}$ | $e_{34}^{(2)} = d_{34}^{(2)} - \Delta b_4^{(2)}$ | $a_3^{(2)} = a_3^{(1)} + \Delta a_3^{(2)} - \Delta m_a^{(2)}$ |
| Prev Effects | $b_1^{(2)} = b_1^{(1)} + \Delta b_1^{(2)} - \Delta m_b^{(2)}$ | $b_2^{(2)} = b_2^{(1)} + \Delta b_2^{(2)} - \Delta m_b^{(2)}$ | $b_3^{(2)} = b_3^{(1)} + \Delta b_3^{(2)} - \Delta m_b^{(2)}$ | $b_4^{(2)} = b_4^{(1)} + \Delta b_4^{(2)} - \Delta m_b^{(2)}$ | $m^{(2)} = m^{(1)} + \Delta m_a^{(2)} + \Delta m_b^{(2)}$ |