Trimmed mean
Mean is too sensitive to extreme observations. Trimmed mean is designed to solve that problem. It involves trimming \( \alpha \) percent observations from both ends. E.g.: If you are asked to compute 10\% trimmed mean, \( \alpha=0.10 \)

Given a bunch of observations, \( X_i \)
1. First, find \( n \) - number of observations
2. Reorder them as "order statistics" \( X_i \) from the smallest to the largest.
3. Compute \( k=n\alpha \) (\( n \) times \( \alpha \))

If \( n\alpha \) is an integer use \( k=n\alpha \) and trim \( k \) observations at both ends. \( R = remaining \ observations = n - 2k=n*(1-2*\alpha) \)

\[
T = \frac{1}{|R|} \sum_{i=g+1}^{n-g-1} X_i
\]

Note: \( \alpha \) is half of the percent of trimming. Mean is \( \alpha = 0\% \) trimmed mean. Median is \( \alpha = CE - CE \cdot ? \)

Example 1: Find 10\% trimmed mean of 2, 4, 6, 7, 11, 21, 81, 90, 105, 121

\( n=10, \alpha=0.10, k=n\alpha=1 \), which is an integer so trim exactly one observation at each end, since \( k=1 \). Thus trim off 2 and 121. we are left with \( R=n-2k=10-2=8 \) observations.

10\% trimmed mean = \((1/8) \cdot (4 + 6 + 7 + 11 + 21 + 81 + 90 + 105)\)

If \( n\alpha \) has a fractional part present, trimmed mean is a bit more complicated. In the above example, if we wanted 15\% trimmed mean, \( \alpha=0.15, n=10, k=n\alpha=1.5 \). Calculations yield \( k \) has an integer part 1, and a fractional part 0.5. \( R=n-2k=10-2*1.5=10-3=7 \). Thus \( R=7 \) observations are retained. This \( R \) is then the correct denominator of trimmed mean.

Since integer part of \( k \) is 1, we throw out the smallest 1 observation, i.e., we throw out \( X_{min}=2 \). The next observation is 4, \( k=1.5 \), has fractional part 0.5, so we throw out only 0.5 part of 4, and keep the remainder 0.5 times 4, or 2. Similarly, at the high end we throw out \( X_{max}=121 \), and throw out 0.5 part of the next largest (=105) and keep the rest (105*0.5)=52.5.

15\% trimmed mean = \((1/7) \cdot ((0.5 \cdot 6) +6 + 7 + 11 + 21 + 81 + 90 + (0.5 \cdot 105))\)

Now find 27\% trimmed mean. \( \alpha=0.27, k=n\alpha=2.7 \), has a fractional part. \( R=n-2k=10-5.4=4.6 \) observations are retained. This \( R \) is the denominator.

27\% trimmed mean = \((1/4.6) \cdot ((0.7 \cdot 6) + 7 + 11 + 21 + 81 + (0.7 \cdot 90))\)

General formula for trimmed mean
\[
T = \frac{1}{n(1 - 2 \cdot \alpha)} \left( (1 - r) * (X_{g+1} + X_{n-g}) + \sum_{i=g+2}^{n-g-1} X_i \right)
\]

\( X \) – sorted numerical vector
\( n \) – length of \( X \)
\( \alpha \) – proportion to trim from each end
\( g \) – integer part of \( n \cdot \alpha \), or floor
\( r \) – fraction part of \( n \cdot \alpha, n \cdot \alpha - g \)