Mann-Whitney test for image segmentation. Non-parametric test

- Define **patch (circle)** and **site (square around it)**.
- Randomly select **Y_1-Y_8 pixels outside of probe site**. Exterior pixels
- Rank order **X_1-X_n probe site pixels**, select **8 smallest**. Interior pixels
- H_0: $\mu_x = \mu_y$, H_1: $\mu_x > \mu_y$
- If not rejected, select next 8 smallest and repeat until rejected.
- Once rejected, these and above are true foreground pixels.

Mann-Whitney U = Wilcoxon signed rank test. Non-parametric

- Let $Y_i = e_i, for\ i = 1, \ldots, m$
- Let $X_i = e_i + \Delta, for\ i = 1, \ldots, n$. H_0: $\Delta = 0$ - **Shift parameter is zero**
- Combine X_i and Y_i – one combined vector
- Order these $m+n$ observation in ascending order. Note R_i to be rank.
- Sum the ranks for X_i, $W = \sum_{i=1}^{n} R_i$ - this is our test statistics.
- Test statistics can be obtained from tables in Hollander-Wolfe "Nonparametric Statistical Methods" book
- Decision rule: One-sided test
    - H_0 vs. H_a that $\Delta > 0$ at α level. Reject H_0 if $W \geq w(\alpha, m, n)$. Or,
    - H_0 vs. H_a that $\Delta < 0$, Reject H_0 if $W \leq (n * (m + n + 1) - w(\alpha, m, n))$. Or,
    - H_0 vs. H_a that $\Delta \neq 0$. Reject H_0 if $W \geq w(\alpha, m, n)\ or \leq \left(n * (m + n + 1) - w(\alpha, m, n)\right)$. Note $\alpha = sum(\alpha_1 + \alpha_2)$ is a sum of alphas for each hypothesis
- Large sample approximation: as *n* and *m* get large, *W** approaches asymptotically standard normal distribution $N(0,1)$. $W^* = \frac{W - (\frac{n(m+n-1)}{2})}{\sqrt{m * n * \frac{m+n-1}{12}}}$.

  Numerator: Observed minus expected; Denominator: Square root of the variance.
- Wilcoxon distribution of U statistics. Relationship between W (rank statistics) and U: $W = U + n * (n + 1)/2$